TOPICS: Finding the zeros of a polynomial function (total, real, complex, and graphically)
End behavior of polynomial graphs
Long and synthetic division of polynomials
Finding all zeros or factors of a polynomial when given a zero or a factor
Complex zeros (occur in pairs)
Rational Zero Test
Intermediate Value Theorem

1. What are the zeros of the function, \( y = k(x) \).

![Graph of y = k(x)](image)

2. What is the maximum number of real zeros that the polynomial function, \( f(x) = 3x^5 - 5x^4 - 4x^3 + 4x^2 - 16x \), may have? *Do not attempt to find the zeros.*

3. For each polynomial function, find all the zeros (real and complex) of each polynomial function, list the multiplicity for each zero, only if greater than one, and determine many zeros will show up on its graph.
   a. \( f(x) = x^4 - 49x^2 \)
   b. \( f(x) = (x + 2)^3(x^2 + 1) \)
   c. \( k(x) = x^3 - 13x^2 + 22x \)

4. Find all the real zeros of each polynomial function.
   a. \( f(x) = x^5 + x^3 \)
   b. \( g(x) = x^4 + x^2 - 12 \)

5. Find a polynomial function that has the given real zeros. *There are multiple correct answers.*
   a. Zeros: \(-2, 3\)
   b. Zeros: 0 (multiplicity 2), \(-4, 5\)
   c. Zeros: \(-3\) (multiplicity 3), \(1\) (multiplicity 2)

6. Find \( k \) such that \( f(x) = x^3 + x^2 - kx - 16 \) has the factor \( x + 4 \).

7. Sketch the graph of the polynomial function using its zeros. Make certain to include tick marks on the \( x \)-axis only (not the \( y \)-axis). Draw long lines. Be accurate and neat, not sloppy.
   a. \( f(x) = -(x + 3)(x + 1)(x - 2) \)
   b. \( g(x) = x(x - 4)^3(x + 2)^2 \)

8. Find a possible function of \( f(x) \). *There are multiple correct answers.*
   a. 
   b. 

9. Is \(-2\) a zero of the polynomial function \( f(x) = x^3 - 6x^2 + 6x - 2 \)? Explain.

10. A zero of a polynomial function of degree 3 with real coefficients is \(-6i\). Name another zero (real or complex).
11. The polynomial function, \( k(x) = x^4 + 6x^3 + 2x^2 + 54x - 63 \), is given.
   a. Find the remaining zeros (real and complex) if \( 3i \) is one of its zeros.
   b. Write the polynomial function \( f(x) \) as the product of factors that are irreducible with integer coefficients.

12. Multiple Choice. Using the Rational Zero Test on polynomial function, \( g(x) = 6x^3 - 4x^2 + 3x - 2 \), how many total possible rational real zeros are there to choose from in helping you determine which, if any, are actual real zeros by a trial-and-error method? Do not attempt to find the zeros.
   a. 36
   b. 12
   c. 8
   d. 4

13. Write the polynomial function, \( k(x) = x^4 + x^3 - 3x^2 - x + 2 \), as the product of irreducible factors with real coefficients.

14. Using the Intermediate Value Theorem, show that the polynomial function \( g(x) = x^3 + 8x^2 + 5x - 14 \) has a zero on the interval \([-1,2]\) and explain why.